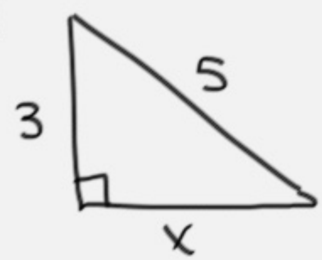
Math 2

Math 1

Study!

Ex



Investigation: Representing Inverse Variation Functions

In this investigation you will explore the key features of an inverse variation function. As you work through the problems look for answers to the following question:

What are the key features of an inverse variation function?
How can the key features be identified using different representations?

NASCAR Racing

Automobile racing is one of the most popular sports in the United States. One of the most important races is the NASCAR Daytona 500, a 500-mile race for cars similar to those driven every day on American streets and highways. The prize for the winner is over $1 million. Winners also get lots of advertising endorsement income.

1. The average speed and time of the Daytona 500 winner varies from year to year.
   a. In 1960, Junior Johnson won with an average speed of 125 miles per hour. The next year Marvin Panch won with an average speed of 150 miles per hour. What was the difference in race time between 1960 and 1961 (in hours)?

   Johnson $= \frac{500}{125} = 4$ hrs. Panch $= \frac{500}{150} = 3.\overline{3}$ hrs. 3hrs 20min

   $\boxed{\text{Difference: } 40\text{min}}$

   b. In 1997, Jeff Gordon won with an average speed of 148 miles per hour. The next year the winner was Dale Earnhardt with an average speed of 173 miles per hour. What was the difference in race time between 1997 and 1998 (in hours)?

   .378 × 60

   Gordon $= \frac{500}{148} = 3.\underline{378}$ 3hrs 23 min

   .890 × 60

   Earnhardt $= \frac{500}{173} = 2.890$ 2hrs 53 min

   $\boxed{\text{difference: } 30\text{min}}$

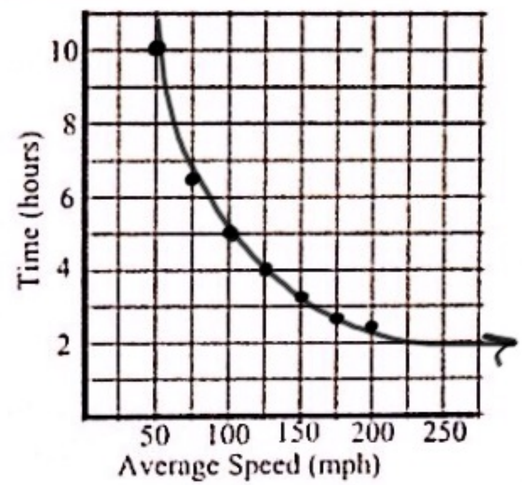
$\frac{500}{50}$

2. Complete a table to display sample pairs of *(average speed, race time)* values for completion of a 500 mile race.

x	Average Speed (mph)	50	75	100	125	150	175	200
y	Race Time (hours)	10	6.67	5	4	3.33	2.86	2.5

a. Plot the sample *(average speed, race time)* data on a graph. Describe the relationship between those two variables.

As avg speed ↑, race time ↓ at a decreasing rate



b. Write a function rule that shows how to calculate the race time *t* as a function of average speed *s* in the Daytona 500 race.

$$\frac{500}{S} = t$$

x y

X In the 1960-61 and 1997-98 comparisons of winning speed and time for the Daytona 500 race, the differences in average speed are both 25 miles per hour. The time differences are not the same. At first, this might seem like a surprising result.

a. How is the fact that equal changes in average speed don't imply equal changes in race time illustrated in the shape of the graph of sample *(average speed, race time)* data?

b. How are the table, graph, and function rule relating average speed and race time similar to or different from those you have seen in work on earlier problems?

4. The function describing race time as a function of the average speed is an example of an inverse variation function.
   a. Below is a partial description of the key features of the function. Use the word bank to complete the description. You may use a word more than once. Also, it is possible that not all of the words will be used.

Description	Word Bank	
The race time is inversely *proportional to the average speed with a constant of proportionality of* 500 miles*. As the average speed increases in* equal intervals *the race time* decreases *at a* decreasing rate*. For this relationship, the speed is* positive *real numbers which means the* domain *is s > 0. The race time is also* positive *real numbers which means the* range *is t > 0.*	*equal intervals*	*500 miles*
	inversely	*directly*
	domain	*range*
	positive	*negative*
	decreases	*increases*
	decreasing rate	*increasing rate*

b. The description above included several key features of the function describing the relationship between average speed and race time. Below is a list of possible key features that may or may not have been included. Identify which ones are included in the description and provide evidence from the description.

Key Feature	Evidence
★★ constant of proportionality	500 miles
★ domain	Positive real #'s $s>0$
★ range	Positive real #'s $t>0$
intercepts	N/A
★★ rate of change	decreasing @ a decreasing rate
intervals of increasing, decreasing, or neither	decreasing @ a decreasing rate
range intervals of positive and/or negative outputs	Positive real #'s
symmetry y	N/A
end behavior	N/A

5. If a car uses g gallons of gasoline in a 200 mile test run, its fuel efficiency is calculated by the formula $E(g) = \boxed{\frac{200}{g}}$ $\frac{k}{x}$. Write a description of the key features of the function $E(g)$.

Constant of proportionality: 200 mile

Domain: $g>0$ (Positive real #s)

Range: $E(g)>0$ (Positive real #'s)

fuel efficiency decreases @ a decreasing rate

6. In both of the contexts (average speed & race time; number of gallons used & fuel efficiency) it can be argued that the domain should have an upper limit. In other words, the domain should not be all positive numbers but should have a maximum value.
    a. Provide a reasoning to support this argument.
    b. What values would make sense in each situation?

The Inverse Variation Function Family

Inverse variation functions share unique characteristics that make them similar to one another and different from other functions. In this section you will explore these unique characteristics using different representations. Use a graphing calculator or computer graphing tool to explore these functions. Desmos. com → start graphing

7. The key features of a function family are often what set them apart from other functions. For each of the inverse variation functions below, identify the key features.

Key Feature	$f(x) = \frac{1}{x}$	$g(x) = \frac{5}{x}$	$h(x) = \frac{-2}{x}$
constant of proportionality	1	5	-2
domain	Positive real #s $x > 0$	→ " "	→ " "
range	Positive real #s $y > 0$	→ " "	negative real #s $y < 0$
intercepts	N/A	→ " "	→ " "
rate of change	decreasing @ a decreasing rate	→ " "	increasing @ a decreasing rate
intervals of increasing, decreasing, or neither	" ↓ "	→ " "	" ↓ "
intervals of positive and/or negative outputs	Positive real #s	→ " "	negative real #s
symmetry	N/A	→ " "	→ " "
end behavior	N/A	→ " "	→ " "

a. Identify the key features of inverse variation functions not in context that are the same regardless of the value of k.

everything is the same except for the ones listed below

b. Identify the key features of inverse variation functions not in context that are effected by the value of k.

Constant of Proportionality, range, rate of change, intervals of incr. decr. or neither

8. How is the shape of the graph for an inverse variation function $y = \frac{k}{x}$ related to the values of k? What happens to the graph as the values of k...

a. increase from 1? moves away from origin

b. between 0 and 1? moves closer to origin

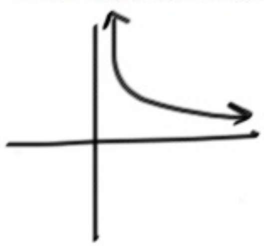
c. are negative? opposite quadrants

orig. → I + III

neg. → II + IV

9. The value of k effects the graph of the function. Compare the following tables of the function $y = \frac{k}{x}$ for different values of k. Explain why the descriptions you determined in problem 5 make sense.

x	$y = \frac{1}{x}$
-2	-0.5
-1	-1
0	Error
1	1
2	0.5



x	$y = \frac{3}{x}$
-2	-1.5
-1	-3
0	Error
1	3
2	1.5

move farther from the origin

x	$y = \frac{0.5}{x}$
-2	-0.25
-1	-0.5
0	Error
1	0.5
2	0.25

moves closer to the origin

x	$y = \frac{-1}{x}$
-2	0.5
-1	1
0	Error
1	-1
2	-0.5

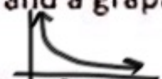
changes from quad I & III & is in quad II & IV now

Check Your Understanding

1. A birthday party at the local climbing wall costs $275 with a limit of 15 guests.
   a. Write a rule relating the *cost per guest* as a function of the *number of guests*.

   $C(g) = \frac{275}{g}$

   b. Make a table and a graph showing *(number of guests, cost per guest)*.

   c. Describe the key features of the function relating *number of guests* and *cost per guest*.

   CoP = 275

   domain = $0 < g \le 15$

   range = $0 < C(g) \le 275$

   $18 < C(g) \le 275$

   rate of change: decreasing @ a decreasing rate