Math 1

★ every friday bring a device to do quizizz on! ★

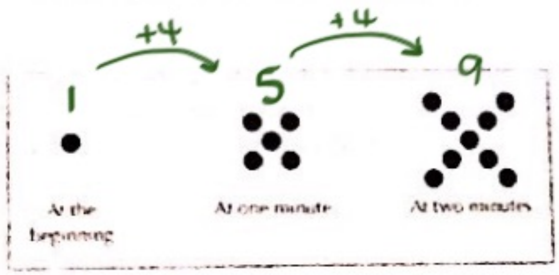
- Take out packet from friday

Lesson: Look For Patterns

Functions include a wide range of situations and can be represented in multiple ways such as graphs, tables, and rules. A **sequence** is a list of numbers or objects that follow a certain pattern. Sequences are considered functions. In this investigation, you will explore different sequences and should look for answers to the following questions

How does the definition of a function apply to sequences?
How are sequences represented by graphs, tables, and rules?
How do we write recursive rules for sequences using subset notation?

Think About This Situation



a) Describe the pattern that you see in the sequence of figures above.

Adding 4 every time

Solve the following problems. Your solution should indicate how many dots will be in the pattern at the given time. Be sure to show how you arrived at your solution and how it relates to the picture.

b) Assuming the pattern continues in the same way, how many dots are there at 3 minutes?

$9+4=\boxed{13}$

c) How many dots are there at 100 minutes?

$4(100)+1 = 401$

d) How many dots are there at *t* minutes?

$y = 4t+1$

Investigation 1: Arithmetic Sequences

The pattern in the *Think About This Situation* represents an **arithmetic sequence**. An arithmetic sequence can be identified by the **constant difference** between consecutive terms.

You will be working with other sequences of numbers that may not fit this pattern, but tables, graphs and equations will be useful tools to represent and discuss these sequences.

1. Complete each table by looking for a pattern.

i.

Term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
Value	2	4	8	16	32	64	128	256

$\times 2$ $\times 2$ $\times 2$ $\times 2$

ii.

Term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
Value	66	50	34	18	2	-14	-30	-46

-16

iii.

Term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
Value	160	80	40	20	10	5	2.5	1.25

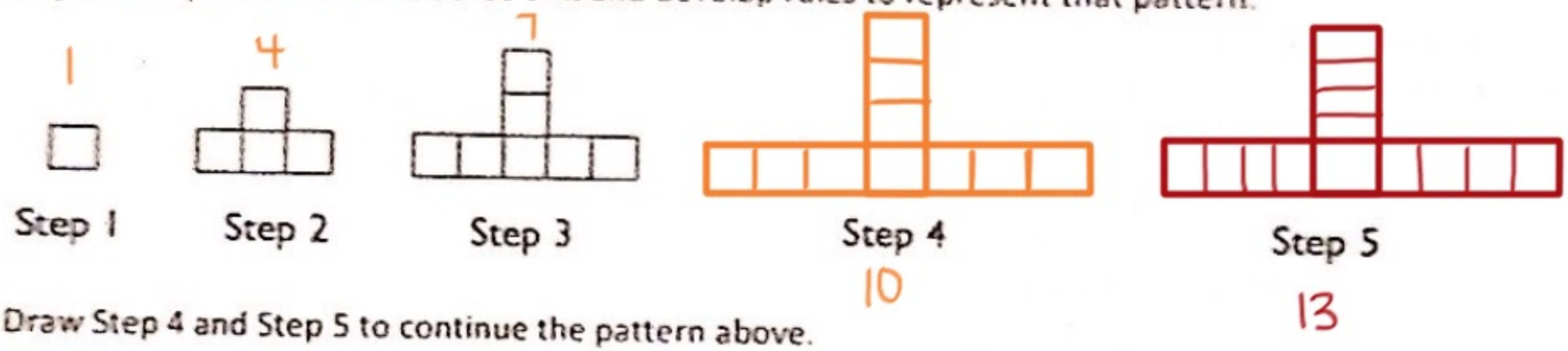
$\times \frac{1}{2}$ $\div 2$

iv.

Term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
Value	-9	-2	5	12	19	26	33	40

$+7$

2. Another sequence represented by a visual pattern is given below. Each square represents one tile. We will investigate this pattern to make sense of it and develop rules to represent that pattern.



i. Draw Step 4 and Step 5 to continue the pattern above.

Add 3

3 The students in Mrs. Holbfield's class were asked to find the number of tiles in a figure by describing how they saw the pattern of tiles changing at each step. Match each student's way of describing the pattern with the appropriate equation below. Note that "s" represents the step number and "n" represents the number of tiles.

Student Descriptions

c Dan explained that the middle "tower" is always the same as the Step number. He also pointed out that the 2 "arms" on each side of the "tower" contain one less block than the Step number.

b Sally counted the number of tiles at each step and made a table. She explained that the number of tiles in each figure was always 3 times the step number minus 2. Her table is below.

Step Number	1	2	3	4	5	6
Number of Tiles	1	4	7	10	13	16

a Nancy focused on the number of blocks in the base compared to the number of blocks above the base. She said the number of base blocks were the odd numbers starting at 1. And the number of tiles above the base followed the pattern 0, 1, 2, 3, 4. She organized her work in the table below.

Step Number	# in Base + # on Top
1	1 + 0
2	3 + 1
3	5 + 2
4	7 + 3
5	9 + 4

Student Equations

a) $n = (2s - 1) + (s - 1)$

b) $n = 3s - 2$ $y = mx + b$

c) $n = s + 2(s - 1)$

4. Can you describe the pattern in your own way, different than Dan, Sally, and Nancy?

Some sequences, like those represented in the previous problems, can be represented by **recursive rules**. A recursive rule is used to determine the next term in a sequence using the previous term.

You may have learned to write a recursive rule as something like NEXT = NOW + 2, starting at 8. Now, we will learn a more formal way to write recursive rules using function notation and subset notation. NEXT = NOW + 2, starting at 8 can be written more formally as:

$$f(n) = f(n-1) + 2,\ f(0) = 8 \quad \text{or} \quad a_{n+1} = a_n + 2,\ a_0 = 8$$

Next = Previous (Now) + 2   x=0 Starting Point

$$\text{or} \quad a_n = a_{n-1} + 2,\ a_1 = 1$$

To think about how to write formal recursive rules, we need to make sense of the notation. We will use tables, graphs, and function notation to make sense of how to write recursive rules more formally than NOW-NEXT form.

5. Finish the table of values for the function represented by the recursive rule NEXT = NOW + 2, starting at 8.

n	0	1	2	3	4	5	6	7	8	9	10
$f(n)$	8	10	12	14	16	18	20	22	24	26	28

+2 +2 +2

i. (A) When $f(n) = 12$, what is the value of n? $n = 2$
(B) What is the value of $f(n-1)$?
$f(2-1) \rightarrow f(1) = 10$

ii. (A) When $f(n) = 14$, what is the value of n? $n = 3$
(B) What is the value of $f(n+1)$?
$f(3+1) \rightarrow f(4) = 16$

iii. (A) When $f(n) = 10$, what is the value of n? $n = 1$
(B) What is the value of $f(n-1)$?
$f(1-1)$
$f(0) = 8$

iv. (A) When $f(n) = 20$, what is the value of n? $n = 6$
(B) What is the value of $f(n+1)$?
$f(6+1) = f(7) = 22$

v. (A) When $f(n) = 22$, what is the value of n? $n = 7$
(B) What is the value of $f(n-6)$?
$f(7-6) = f(1) = 10$

vi. (A) When $f(n) = 16$, what is the value of n? $n = 4$
(B) What is the value of $f(n+3)$?
$f(4+3)$
$f(7) = 22$.

6. So far, we have used vocabulary like Now, Next, term, value, and starting value to communicate about sequences and recursive rules. Use this vocabulary to describe what each of the following notations represent:

i. $f(n) \rightarrow$ Now

ii. $f(n+1) \rightarrow$ Next

iii. $f(n-1) \rightarrow$ Previous

iv. $f(0) \rightarrow$ Starting point
$x = 0$

Function notation is only one way to formally communicate terms in a recursive rule. Another formal way is **subset notation**. Below, both rules, in subset notation, represent the same recursive rule we've been working with:

NEXT = NOW + 2, starting at 8

x=0 start term

$a_{n+1} = a_n + 2, \quad a_0 = 8$

Next   Now

$a_n = a_{n-1} + 2, \quad a_1 = 10$

Now   Previous   1st term

7. What is different about the two recursive rules in subset notation? What is the same?

Next # compared to Now / Add 2 every time

8. Thinking about the meaning of the function notation you worked with in #6, what do you think each of the following notations mean?

i. a_{n+1} Next # (next x-value up)

ii. a_n Now

iii. a_{n-1} Previou (the x-value before)

iv. a_0 Starting point (x=0) term 0

v. a_1 1st term

9. Write a function rule to represent the recursive rule we've been using: NEXT = NOW + 2, starting at 8

i. Function rule: $f(x) = 2x + 8 \rightarrow$ Start. $m = \frac{2}{1}$

$y = mx + b$ ↳ Changing by

ii. A few terms and corresponding values of this function as a sequence are 8, 10, 12, 14, 16. Sketch a graph of the sequence.

Term #	0	1	2	3	4	...
Value	8	10	12	14	16	...

iii. What subset notation represents the starting value, 8, or $f(0) = 8$? This is not considered the "first term". Why? this is term 0 or a_0 not the first term or a_1

iv. What subset notation would represent the first term, 10, or $f(1) = 10$?

a_1

10. Match the following explicit rules, recursive rules, and tables.

Explicit Rules

A. $f(x) = 3x + 7$

Now + 3 start 7

B. $f(x) = 10x - 2$

+ 10 start

C. $f(x) = 5(2^x)$

start ×2

D. $f(x) = -2x + 3$

−2 start

Recursive Rules (~~function notation~~)

I. $f(n) = 2f(n-1),\ f(1) = 10$

II. $f(n) = f(n-1) + 10,\ f(1) = 8$

III. $f(n) = f(n-1) - 2,\ f(1) = 1$

IV. $f(n) = f(n-1) + 3,\ f(1) = 10$

A	IV	M	c
B	II	K	a
C	I	L	b
D	III	J	d

Recursive Rules (subset notation)

J. $a_n = a_{n-1} - 2,\ a_1 = 1$

K. $a_n = a_{n-1} + 10,\ a_1 = 8$

L. $a_n = 2a_{n-1},\ a_1 = 10$

M. $a_n = a_{n-1} + 3,\ a_1 = 10$

Tables

a.

Term #	1	2	3	4
Value	8	18	28	38

b.

Term #	1	2	3	4
Value	10	20	40	80

c.

Term #	1	2	3	4
Value	10	13	16	19

d.

Term #	1	2	3	4
Value	1	-1	-3	-5

11 One of the above rules is not considered arithmetic. Which one? Explain why you think this.

$y=5(2^x)$ → exponential so not increasing/decreasing by a constant rate.

12 Find the next 3 terms in each arithmetic sequence. Identify the constant difference. Write a recursive function in subset notation and an explicit function for each sequence. Circle where you see the common difference in each function. (Note: the first number of the sequence is the 1st term, not the 0th term.)

0 1 2 3 4 5

i. -2 3, 8, 13, 18, 23, 28, 33, 38

-5 +5

Common difference: +5

Recursive function: $a_n = a_{n-1} + 5$, $a_1 = 3$

$y = mx + b$

Explicit function: $f(x) = 5x - 2$

0 1 2 3

ii. 13 11, 9, 7, 5, 3, 1, -1, -3

+2 -2

Common difference: -2

Recursive function: $a_n = a_{n-1} - 2$, $a_1 = 11$

Explicit function: $f(x) = -2x + 13$

0 1

iii. 4.5 3, 1.5, 0, -1.5, -3, -4.5, -6, -7.5

+1.5 -1.5

Common difference: -1.5

Recursive function: $a_n = a_{n-1} - 1.5$, $a_1 = 3$

Explicit function: $f(x) = -1.5x + 4.5$

13. Below you are given various types of information. Write the recursive rule (in subset notation) and explicit rule for each arithmetic sequence. Finally, graph each sequence, making sure you clearly label your axes.

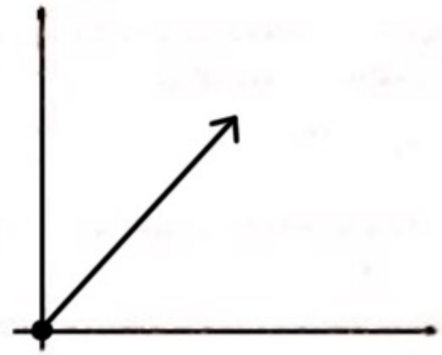
0 1 2 3 4

i. 0, 2, 4, 6, 8, ...

-2 +2

Recursive: $a_n = a_{n-1} + 2$, $a_1 = 2$

Explicit: $f(x) = 2x$



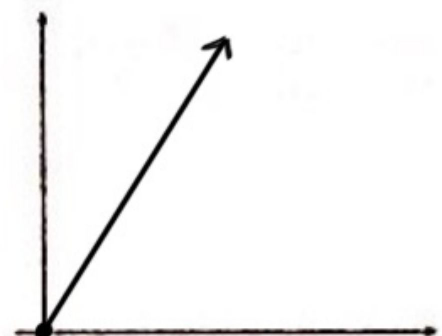
ii.

Time (Days)	0	1	2	3	4
Number of Cell	0	3	6	9	12

-3 +3 +3 +3

Recursive: $a_n = a_{n-1} + 3$, $a_1 = 3$

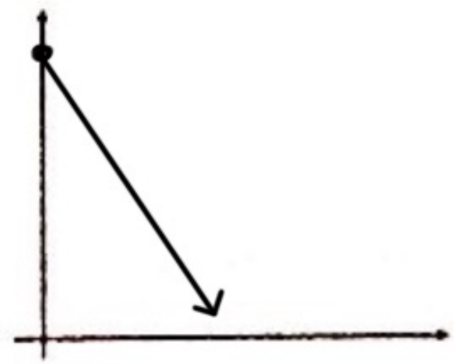
Explicit: $f(x) = 3x$



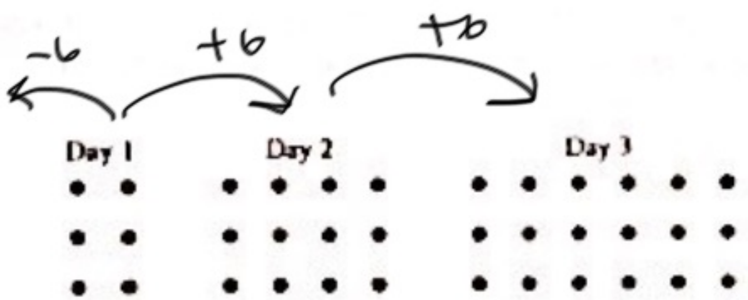
iii. Claire has $300 $\rightarrow a_0$ in an account. She decides she is going to take out $25 each month -25

Recursive: $a_n = a_{n-1} - 25,\ a_1 = 275$

Explicit: $f(x) = -25x + 300$



iv. 0



Recursive: $a_n = a_{n-1} + 6,\ a_1 = 6$

Explicit: $y = 6x$

$\downarrow$

$f(x) = 6x$