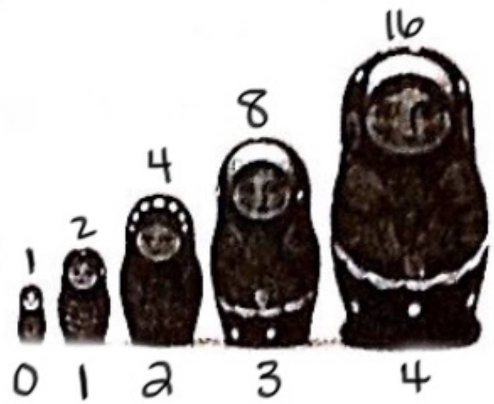
Math 1

Take out
HW

D.1 : EXPONENTS AND THEIR PROPERTIES

Think About the Situation

A *matryoshka* doll, also known as Russian nesting doll, refers to a set of wooden dolls of decreasing size placed one inside the other.



In a set of nesting dolls, the smallest doll, Doll 0, is 1 inch tall. Suppose each doll is twice as tall as the previous doll. How tall is each of the following dolls?

a. Doll 1 2
b. Doll 3 $8 = 2 \cdot 2 \cdot 2 = 2^3$
c. Doll 8 $256 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^8$
d. Doll *n* 2^n

Investigation 1: Exponents and Their Properties

Perhaps you first used exponents to find areas and volumes. For example, the formula for the area of a square with side length *s* is $A = s^2$, often read as "s squared." The volume of a cube with side length s is $V = s^3$, often read as "s cubed."

There are other applications for the use of exponents. As you have already experienced, exponents can be used in expressions to represent growth. They can also be used in expressions to represent decay. In solving problems involving exponential expressions, it helps to know some basic methods for reasoning with and writing the expressions in equivalent forms.

As you work on problems in this investigation, look for answers to this question:

How can the definition of exponent be used to discover and justify other properties of exponents that make useful algebraic manipulations possible?

Definition of an exponent

Exponents are used to show *repeated multiplication*. For example, you can write $3 \times 3 \times 3 \times 3$ as 3^4. You need to have an understanding of the parts of an exponential expression and their meaning. This will allow you to make calculations, simplify expressions, and explain and apply the basic rules of exponents.

Let's look at the parts of the expression 3^4.

The 3 is the base of the expression → 3^4 ← The 4 is the exponent of the expression.

The expression 3^4 means that there are "4 factors of 3".

$$3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{\text{4 factors of 3}}$$

1. Interpret the meaning of each of the following expressions.

a. 2^8 - multiply 2 eight times - 8 factors of 2
b. 10^5 mult 10 five times
c. 6^1 mult 6 once
d. x^3 3 factors of x
e. $4^3 \cdot 5^2$ 3 factors of 4 + 2 factors of 5
f. x^3y^1 mult x three times + mult y once
g. x^1 mult x once
h. $(5x)^2$ mult 5x twice

Products of Powers Work with exponents is often helped by writing products like $b^r \cdot b^s$ in simpler form or by breaking a calculation like b^t into a product of two smaller numbers.

2. Find values for w, x, and y that will make these equations true statements:

a. $2^{10} \cdot 2^3 = 2^x$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{13}$

b. $5^2 \cdot 5^4 = 5^x$ 5^6

c. $3^1 3^7 = 3^y$ $= 3^8$

d. $2^w \cdot 2^4 = 2^7$ $w = 3$

e. $b^4 \cdot b^2 = b^x$

b^6

f. $9^w \cdot 9^x = 9^5$

$9^2 \cdot 9^3$ $9^5 \cdot 9^0$
$9^1 \cdot 9^4$

3. Examine the results of your work on Problem 2.

a. What pattern seems to relate task and result in every case? Adding the exponents

b. Provide a convincing argument using the definition of exponent to justify your conclusion in part a.

$$x^a \cdot x^b = x^{a+b}$$

4. Simplify each of the following

a. $(x^2y^4)(3x)$ $\boxed{3x^3y^4}$

b. $m^5n^2 \cdot m^7n^1 \cdot m^4n^4$ $m^{16}n^7$

c. $(2a^1b^5)(5a^2)(a^1b^8)$ $\boxed{10a^4b^{13}}$

d. $\left(\frac{3x^1}{y^2}\right)\left(\frac{5x^3}{y^5}\right) = \boxed{\frac{15x^4}{y^7}}$

Power of a Power You know that $8=2^3$ and $64=8^2$, so $64=(2^3)^2$. As you work on the next few problems, look for a pattern suggesting how to write equivalent forms for expressions like $(b^x)^y$ that involve powers of powers.

5. Find values for x and z that will make these equations true statements.

a. $(2^x)^4 = 2^z$

$(2\cdot2\cdot2)(2\cdot2\cdot2)(2\cdot2\cdot2)(2\cdot2\cdot2) = 2^{12}$

b. $(3^5)^2 = 3^z$ 3^{10}

$(3\cdot3\cdot3\cdot3\cdot3)(3\cdot3\cdot3\cdot3\cdot3)$

c. $(5^x)^3 = 5^z$

$(5^2)^3 = 5^6$

d. $(b^2)^5 = b^z$

b^{10}

6. Examine the results of your work on Problem 5.

a. What pattern seems to relate task and result in every case? multiply the exponents

b. Provide a convincing argument using the definition of exponent to justify your conclusion in part a. $(x^a)^b = x^{ab}$

7. Simplify each of the following

a. $-3(x^3)^2$

$\boxed{-3x^6}$

b. $(3y^5)^2(2xy^2)$

$(3)^2(y^5)^2 = (9y^{10})(2x^1y^2) = \boxed{18xy^{12}}$

c. $(-5u^4)^2$

$(-5)^2(u^4)^2$

$\boxed{25u^8}$

d. $\left(\frac{2y^5}{3x^2}\right)^3$

$\frac{(2)^3(y^5)^3}{(3)^3(x^2)^3} = \boxed{\frac{8y^{15}}{27x^6}}$

1

a. $9x^1 \cdot x^5$ $\boxed{9x^6}$

b. $5x^2 \cdot 4x^3$

c. $(-8x^4)(-3x^1)$ $\boxed{24x^5}$

d. $(6x^4)(-x^6)$

K $24x^5$

O $-6x^{10}$

A $9x^6$

E $24x^{10}$

L $20x^5$

2

a. $(7n^3)^2$

$(7)^2(n^3)^2 = \boxed{49n^6}$

b. $(-4n^8)^3$

c. $(5n^4)^4$

$(5)^4(n^4)^4 = \boxed{625n^{16}}$

d. $(-2n^2)^6$

W $64n^{12}$

I $625n^{12}$

U $-64n^{24}$

B $49n^6$

D $625n^{16}$

3

a. $(4m^7d^2)^2$

$(4)^2(m^7)^2(d^2)^2 = \boxed{16m^{14}d^4}$

b. $(-9m^4d^3)^2$

c. $(-1m^2d^5)^3$

$(-1)^3(m^2)^3(d^5)^3 = \boxed{-1m^6d^{15}}$

$\boxed{-m^6d^{15}}$

d. $(-3md^9)^4$

T $16m^{14}d^4$

G $-m^6d^{15}$

V $81m^8d^6$

E m^6d^8

R $81m^4d^{36}$

4

$(x^2)^2$

a. $3xy(5x^2y)^2$

$(3xy)(25x^4y^2) = \boxed{75x^5y^3}$

b. $(-7y)(2xy^2)^3$

$(-10)^2 (x^5)^2 (y^4)^2$

c. $x^8y^3(-10x^5y^4)^2$

$(x^8y^3)(100x^{10}y^8)$ $\boxed{100x^{18}y^{11}}$

d. $(xy^4)^4(-9y^3)$

S $-56x^3y^7$

N $100x^{20}y^7$

C $75x^5y^3$

F $-9x^4y^{19}$

T $100x^{18}y^{11}$

Quotients of Powers Since many useful algebraic functions require division of quantities, it is helpful to be able to simplify expressions involving quotients of powers like $\frac{b^x}{b^y}$ $(b \neq 0)$.

8. Find values for x, y, and z that will make these equations true statements.

a. $\frac{2^{10}}{2^3} = 2^x = 2^7$

$\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}}$

b. $\frac{3^6}{3^2} = 3^y$

$3^{6-2} = 3^4$

c. $\frac{10^9}{10^3} = 10^z$

10^6

d. $\frac{2^x}{2^5} = 2^7$

2^{12}

e. $\frac{7^x}{7^y} = 7^2$

$x = 6$ $\quad x = 10$

$y = 4$ $\quad y = 8$

f. $\frac{b^5}{b^3} = b^z$

b^2

g. $\frac{3^5}{3^5} = 3^z$

$3^0 = 1$

h. $\frac{b^x}{b^x} = b^z$

$b^0 = 1$

9. Examine the results of your work on Problem 8.

a. What pattern seems to relate task and result in every case? Subtracting Numerator − denom.

b. Provide a convincing argument using the definition of exponent to justify your conclusion in part a.

c. Explain why it is reasonable to define $b^0 = 1$ for any base b $(b \neq 0)$?

$\frac{x^a}{x^b} = x^{a-b}$

10. Simplify each of the following.

a. $\frac{27m^5n^4}{9mn} = \boxed{3m^4n^3}$

b. $\frac{(-2x^2y)^3}{9x^2y^2} = \frac{-8x^6y^3}{9x^2y^2} = \boxed{\frac{-8x^4y}{9}}$

$(-2)^3(x^2)^3(y)^3$

c. $\frac{14a^5b^3}{7a^2b^2} = \boxed{2a^3b}$

d. $\frac{x^{2a}y^a}{x^ay}$

Exponent Property	Example	Words
Exponent of one	$x^1 = x$	Any thing to the power of **ONE** is itself. It means there is only "one factor of that base".
Zero Exponents	$x^0 = 1$ $x \neq 0$	Any thing to the **ZERO** power will **always** be ONE! Except a base of 0... that doesn't work
Product Property	$x^a \cdot x^b = x^{a+b}$	When **multiplying** with exponents, you **add** them as long as the **bases are the same!**
Quotient Property	$\frac{x^a}{x^b} = x^{a-b}$	When **Dividing** with exponents, you **subtract** them as long as the **bases are the same!** I ask myself, "Which has more... the top or bottom? How many more?"
Power to a Power	$(x^a)^b = x^{a \cdot b}$	When you have a power raised to another **power**, you **multiply** them!

Negative Exponents	$x^{-a} = \frac{1}{x^a}$ $\frac{1}{x^{-a}} = x^a$	When you have a **negative** exponent it **moves** it to the **denominator** and drops the negative. If its already in the denominator, you move it to the numerator!
		When you have an **exponent** that is a **fraction**, you can convert it to ~~a radical expression.~~ The **numerator** is the base's exponent ~~**on the inside**~~ and the **denominator** is the index on the **outside**!

★ Remember: You can always expand ex) $x^3 = x \cdot x \cdot x$

★ Remember: These 2 Are Very different ex) $2x^2 = 2 \cdot x \cdot x$

$(2x)^2 = (2 \cdot x)(2 \cdot x)$