Math I

take out
Packet from
thursday
↳ Exponential
Notes.

Test
Wednesday

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Exponential Decay

Equation

$y = a(b^x)$

Start value

multiplier

↳ $0 < b < 1$ decimal or fraction

100%

$y = a(1 - r)^x$

start value

% rate as a Decimal

Table

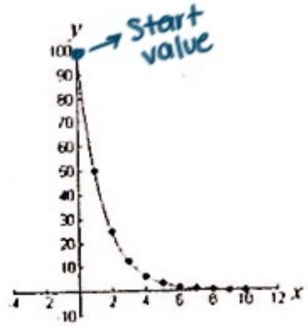
$y = 100(\frac{1}{2})^x$

.50

50% decay

X	Y
0	100
1	50
2	25
3	12.5
4	6.25
5	3.125

Graph



Words

The population of an endangered species is cut in half each year. The current population is 2000. Write an equation

rate 50% decay.

Start value

$y = 2000(1 - .5)^x$

$\boxed{y = 2000(.5)^x}$

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Modeling Exponential Functions

Penicillin was discovered by observing mold on biology lab dishes. When first observed, the mold covered 7 cm^2 of the dish surface. It then appeared to triple in area every day.

a. Fill in the table below for the area of the mold patch y for every day x.

x	0	1	2	3	4	5	6	7 (one week)
y	7	21	63	189	567	1701	5103	15309

x3 x3 x3

b. Write a now-next rule.

Next = 3·Now starting at 7

c. Write an equation that would model the problem above.

$y = 7(3^x)$

d. How would the equation change if there were 5cm of mold at the beginning of the observation? If it doubled every day?

$y = 5(2^x)$

Analyzing Growth and Decay Models

Determine the multiplier for each growth or decay rate.

a. 5% growth
.05
$1 + .05$
$\boxed{1.05}$

b. 12% decay
.12
$1 - .12$
$\boxed{.88}$

c. 30% growth
.30
$1 + .30$
$\boxed{1.30}$

d. 2.5% decay
.025
$1 - .025$
$\boxed{.975}$

State whether the formula models growth or decay.

a. $y = 3^x$ ← 300%
growth 200%

b. $y = 0.25^x$ $1 - r = .25$
.75
Decay 75%

c. $y = 500(0.97)^x$
Decay 3%

d. $y = 6(5)^x$ → 500%
growth 400%

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Modeling Growth and Decay

In 2013 the population of Asheville was 87,236. If it has been increasing by 6% every year, how many people live in Asheville today (in 2016)?

Formula:

$y = a(1+r)^x$

$y = P(1+r)^t$

$P = 87{,}236$ $r = .06$ $t = 3$

$y = 87{,}236(1+.06)^3$

$\boxed{y = 87{,}236(1.06)^3}$

There are huge colonies of ants that only eat a certain type of fungus that they cultivate in their colonies. If their mold supply covers 2,500 in^2 and they eat it at a rate of 6% per week, how much would remain in 7 weeks?

Formula:

$y = a(1-r)^x$

$y = P(1-r)^t$

$P = 2500$ $r = .06$ $t = 7$

$y = 2500(1-.06)^7$

$\boxed{y = 2500(.94)^7}$

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Math I – Notes — Analyzing Exponential Tables

Example 1: *Analyzing Tables Exponential Tables*

a. Exponential Growth

x	0	1	2
y	1000	1050	1102.50

50

What is the initial value (y-intercept)?

1000

What is the growth rate?

What % of 1000 is 50

$\frac{50}{1000} = .05$ or 5% $\boxed{\text{growth } \%}$

$1 + .05$

$\boxed{1.05}$ growth rate

Next = Now

Next = 1.05 · Now starting at 1000

y =

$y = 1000(1.05)^x$

$y = 1000(1 + .05)^x$

b. Exponential Decay

x	0	1	2
y	1000	800	640

200

What is the initial value (y-intercept)?

1000

What is the growth rate?

$\frac{200}{1000} = .20$ or 20%

Decay % → 20%

$1 - .20$

$\boxed{.80}$

Next = Now

Next = .80 · Now Starting at 1000

y =

$y = 1000(.80)^x$

$y = 1000(1 - .20)^x$

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xample 2: ***Application to Table of Values***

Decay % $= \frac{1}{3} = \boxed{.33}$ or 33%

growth rate (multiplicative rate) : $1 - .33 = \boxed{.67}$

① Suppose a new golf ball drops downward from a height of 27 feet onto a paved parking lot and keeps bouncing up and down, again and again. Rebound height of the ball should be $\frac{2}{3}$ of its drop height. Make a table and plot of the data showing expected heights of the first ten bounces of the golf ball.

Bounce Number	0	1	2	3	4	5	6	7	8	9	10
Rebound Height (in feet)	27	18	12	8	5.3	3.6	2.4	1.6	1.1	.7	.5

x .67 .67

2/3

~~a.~~ **Check your accuracy by calculating the remaining fraction after each bounce? Check at least points to ensure accuracy.**

b. Write a rule using the original height and the decay factor.

$$y = 27(1 - .33)^x$$

$$\boxed{y = 27(.67)^x}$$

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Jse the table below to answer the following questions:

$\% = \frac{\text{Change}}{\text{original}} = \frac{200}{800} = .25$

x	0	1	2	3	4	5	6
y	800	600	450	337.5	253.1	189.8	

200

a. What is the fraction or percentage remaining for each step?

Decay % = .25 or 25%

multiplicative rate (growth rate) : $1 - .25 = \boxed{.75}$

b. What is the original amount (y-intercept)?

800

c. What is the rule for the table above?

$$\boxed{y = 800(.75)^x}$$

$$y = 800(1 - .25)^x$$

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Guided Practice Problems –

1. When dropped onto a hard surface, a brand new softball should rebound to about $\frac{3}{5}$ the height from which it is dropped.

$\text{Decay \%} = \frac{3}{5} = .60$

a. If the softball is dropped 25 feet from a window onto concrete, what pattern of rebound heights can be expected?

growth rate (multiplicative rate): $1 - .60 = .40$ ← multiplier

i. Make a table and plot of predicted rebound data for 5 bounces.

x	0	1	2	3	4	5
y	25	10	4	1.6	.64	.256

$\times .4 \quad \times .4 \quad \times .4$

b. Write a rule for the table above.

$$\boxed{y = 25(.40)^x}$$

$$y = 25(1 - .60)^x$$

2. Write a rule for each of the tables below

a.

x	0	1	2	3	4	5
y	10	9	8.1	7.29	6.561	5.9049

$\text{decay \%} = \frac{1}{10} = .10$

$\text{multiplicative rate} = 1 - .10 = .90$

$$y = 10(.90)^x$$

b.

x	0	1	2	3	4	5
y	11.7	3.8	1.3	0.6	0.2	0.05

$\div .34 \quad \times .34 \quad \times .34 \quad \times .34$

$\frac{2.5}{3.8} = .66$

growth $1 - .66 = .34$

$$\boxed{y = 11.7(.34)^x}$$

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1. $3y^{-5}\cdot 3y^4 = \frac{3\cdot 3y^4}{y^5} = \frac{9y^4}{y^5} = \frac{9\cancel{y}\cancel{y}\cancel{y}\cancel{y}}{\cancel{y}\cancel{y}\cancel{y}\cancel{y}y} = \boxed{\frac{9}{y}}$

2. $5a^7b^{-5}\cdot 2a^{-5}b^5 = \frac{5a^7\cdot 2b^5}{b^5a^{+5}} = \frac{10a^7b^5}{a^5b^5} = 10a^2\underset{1}{b^0} = \boxed{10a^2}$

3. $(3a^0)^3$

$(3)^3(a^0)^3 = (3)^3(1)^3 = 27\cdot 1 = \boxed{27}$

4. $(5m^2)^{-3}$

$(5)^{-3}(m^2)^{-3} = 5^{-3}m^{-6} = \frac{1}{5^3m^6} = \boxed{\frac{1}{125m^6}}$

5. $(2z^3\cdot z\cdot 3)^3$

$(2)^3(z^3)^3(z)^3(3)^3$

$8\cdot z^9\cdot z^3\cdot 27$

$\boxed{216z^{12}}$

6. $\frac{5a^5}{5a^4} = \boxed{a^1}$

7. $\frac{7r^6s^{-7}t^5}{8st} = \frac{7r^6t^5}{8sts^7}$

$= \frac{7r^6t^5}{8s^8t} = \boxed{\frac{7r^6t^4}{8s^8}}$

8. $\left(\frac{3r^2k^6}{8r^4k^3}\right)^3 = \frac{(3)^3(r^2)^3(k^6)^3}{(8)^3(r^4)^3(k^3)^3} = \frac{27r^6k^{18}}{512r^{12}k^9} = \boxed{\frac{27k^9}{512r^6}}$

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9. $\dfrac{4g^5d^3}{(5gd^4)^{-2}} = \dfrac{4g^5d^3}{(5)^{-2}(g)^{-2}(d^4)^{-2}} = \dfrac{4g^5d^3}{5^{-2}g^{-2}d^{-8}} = 4g^5d^3 \cdot 5^2g^2d^8 = \boxed{100g^7d^{11}}$

10. $\dfrac{5a^{-3}}{a^6} = \dfrac{5}{a^6a^3} = \boxed{\dfrac{5}{a^9}}$

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