Math 1

- Always get your calculators from now on!
- Take out Packet from yesterday
- need TOOLKITS & gluesticks

Math I Investigation 5.1.2 Getting Started **Name:** ______________________

The patterns of change that occur in counting the good deeds of a Pay It Forward scheme and the growing number of bacteria in a cut are examples of *exponential growth*. Exponential functions get their name from the fact that in rules like $N = 2^x$ and $N = 3^x$, the independent variable occurs as an exponent. As you work on the problems in this investigation, look for answers to these questions:

What are the forms of NOW-NEXT and "y = ___" rules for basic exponential functions?
How can those rules be modified to model other similar patterns of change?

Infections seldom start with a single bacterium. Suppose that you cut yourself on a rusty nail that puts 25 bacteria cells into the wound. Suppose also that those bacteria divide in two after every quarter of an hour.

a. Make and record a guess of how many bacteria you think would be present in the cut after 8 hours (32 quarter-hours) if the infection continues to spread as predicted. (Assume your body does not fight off the infection and that you do not apply medication.) Then answer the following questions to check your ability to estimate the rate of exponential growth.

Guess: ____________

b. Complete this table showing the first several numbers in the bacteria growth pattern:

Number of Quarter-Hour Periods	0	1	2	3	4
Number of Bacteria in the Cut	25	50	100	200	400

x2

c. Use *NOW* and *NEXT* to write a rule showing how the number of bacteria changes from one quarter-hour to the next, starting from 25 at time 0.

Next = 2 · Now starting at 25

d. Write a rule showing how to calculate the number of bacteria *N* in the cut after *x* quarter-hour time periods.

$N = 25(2^x)$

e. Use the rules in parts c and d to calculate the number of bacteria after 8 hours. Then compare the results to each other and to your initial estimate in Part a.

↓ 32 quarter hours

$N = 25(2^{32})$

101,374,182,400 bacterium

10^{11}

1.073741824 E11

2. Compare the pattern of change in this situation to the simple case that started from a single bacterium by noting similarities and differences in:

a. tables of (*number of time periods, bacteria count*) values

same	different
· multiplied by 2	· starting values

25
Next = 2 · Now starting @25

1
Next = 2 · Now

b. graphs of (*number of time periods, bacteria count*) values

Same	difference
· shape	· starting value (y-intercept)
	· rate of change.

25
$y = 25(2^x)$

1
$y = 2^x$

c. NOW-NEXT and "N = ..." rules

same	different
multiplying it by 2	Starting values.

3. Investigate the number of bacteria expected after 8 hours if the starting number of bacteria is 30, 40, 60, or 100, instead of 25. For each starting number at time 0, complete parts a – c. Work with your partner to complete the work.

2 hr

a. Make a table of (*number of time periods, bacteria count*) values for 8 quarter-hour time periods.

# Quarter Hours	0	1	2	3	4	5	6	7	8
# Bacteria in Cut	30	60	120	240	480	960	1920	3840	7680

x2 x2

# Quarter Hours	0	1	2	3	4	5	6	7	8
# Bacteria in Cut	40	80	160	320	640	1280	2560	5120	10240

# Quarter Hours	0	1	2	3	4	5	6	7	8
# Bacteria in Cut	60	120	240	480	960	1920	3840	7680	15360

# Quarter Hours	0	1	2	3	4	5	6	7	8
# Bacteria in Cut	100	200	400	800	1600	3200	6400	12800	25600

b. Write two rules that model the bacteria growth – one relating *NOW* and *NEXT* and the other beginning "*N* = ..."

Rules for 30:

Next = 2·Now starting at 30

$N = 30(2^x)$

Rules for 40:

Next = 2·Now starting at 40

$N = 40(2^x)$

Rules for 60:

Next = 2·Now starting at 60.

$N = 60(2^x)$

Rules for 100:

Next = 2·Now starting 100

$N = 100(2^x)$

c. Use each rule to find the number of bacteria after 8 hours and check to see that you get the same result.

Original # Bacteria	30	40	60	100
~~NOW-NEXT at 8 hours~~				
$N = \ldots$ at 8 Hours ↓ 32 quarter hours	1.29×10^{11}	1.72×10^{11}	2.58×10^{11}	4.29×10^{11}

$N = 30(2^{32})$ $N = 40(2^{32})$

d. Now compare results from two of the cases – starting at 30 and starting at 40.

i. How are the *NOW-NEXT* and the "N = ..." rules similar and how are they different?

ii. How are the patterns in the tables and graphs of (*number of time periods, bacteria count*) data similar and how are they different?

Just as bacteria growth won't always start with a single cell, other exponential growth processes can start with different initial numbers. Think again about the Pay It Forward scheme in Investigation 1.

4. Suppose that four good friends decide to start their own Pay It Forward tree. To start the tree, they each do good deeds for 3 different people. Each of those new people in the tree does good deeds for 3 other new people, and so on.

not ppl

a. What *NOW-NEXT* rule shows how to calculate the number of good deeds done at each stage of this tree?

Next = 3·Now starting at 12

0	1	2
4	12	36

b. What "*N* = ..." rule shows how to calculate the number of good deeds done at any stage *x* of this tree?

$N = 4(3^x)$

c. How would the *NOW-NEXT* and "*N* = ..." rules be different if the group of friends starting the tree had 5 members instead of 4?

Next = 3·Now Starting at 15

$N = 5(3^x)$

d. Which of the Pay It Forward schemes below would most quickly reach a stage in which 1,000 good deeds are done? Why does that make sense?

Scheme 1: Start with a group of 4 friends and have each person in the tree do good deeds for 2 different people; or $N = 4(2^x)$ $x = 8$

Scheme 2: Start with only 2 friends and have each person in the tree do good deeds for 3 other people.

$N = 2(3^x)$ $x = 6$

In studying exponential growth, it is helpful to know the initial value of the growing quantity. For example, the initial value of the growing bacteria population in problem 1 was 25. You also need to know when the initial value occurs. For example, the bacteria population was 25 after 0 quarter-hour period.

In problem 4, on the other hand, 12 good deeds are done at Stage 1. In this context, "stage 0" does not make much sense, but we can extend the pattern backward to reason that $N = 4$ when $x = 0$.

5. Use your calculator and the ^ key to find each of the following values: $2^0, 3^0, 5^0, 23^0$.

a. What seems to be the calculator value for b^0, for any positive value of b?

always 1

b. Recall the examples of exponential patterns in bacterial growth. How do the "$N = ...$" rules for those situations make the calculator output for b^0 reasonable?

6. Now use your calculator to make tables of (x, y) values for each of the following functions. Use integer values for x from 0 to 6. Make notes of your observations and discussion of questoins in Parts a and b

i. $y = 5(2^x)$ ii. $y = 4(3^x)$ iii. $y = 3(5^x)$ iv. $y = 7(2.5^x)$

y= → type the function

X =	0	1	2	3	4	5	6
$y = 5(2^x)$	5	10	20	40	80	160	320
$y = 4(3^x)$	4	12	36	108	324	972	2916
$y = 3(5^x)$	3	15	75	375	1875	9375	46875
$y = 7(2.5^x)$	7	17.5	43.75	109.38	273.44	683.59	1708.98

a. What patterns do you see in the tables? How do the patterns depend on the numbers in the function rule?

b. What differences would you expect to see in tables of values and graphs of the two exponential functions $y = 3(6^x)$ and $y = 6(3^x)$?

start lower

increase faster

start higher

increase slower than the multiplier of 6.

7. Suppose you are on a team studying the growth of bacteria in a laboratory experiment. At the start of your work shift in the lab, there are 64 bacteria in one petri dish culture, and the population seems to be doubling every hour.

a. What rule should predict the number of bacteria in the culture at a time x hours after the start of your work shift?

$$N = 64(2^x)$$

b. What would it mean to calculate values of y for negative values of x in this situation?

neg hours are hours in the past

0 hrs	12 pm
-1 hrs	11 am
-2 hrs	10 am

c. What value of y would you expect for $x = -1$? For $x = -2$? For $x = -3$ and -4?

~~-7~~	-6	-5	-4	-3	-2	-1	0	1
~~.5~~	1	2	4	8	16	32	64	128

d. Use your calculator to examine a table of (x, y) values for the function $y = 64(2^x)$ when $x = 0, -1, -2, -3, -4, -5, -6$. Compare results to your expectations in Part c. Then expalin how you could think about this problem of bacteria growth in a way so that the calculator results make sense.

8. Study tables and graphs of (x, y) values to estimate solutions for each of the following equations and inequalities. In each case, be prepared to explain what the solution tells about bacteria growth in the experiment of Problem 7.

a. $1.024 = 64(2^x)$

$x = -4$

b. $8,192 = 64(2^x)$

$x = 7$

c. $64(2^x) > 25,000$

$x \geq 9$

d. $4 = 64(2^x)$

$x = -4$

e. $64(2^x) < 5,000$

$x < 6$

f. $64(2^x) = 32$

$x = -1$

EXPONENTIAL GROWTH

EQUATION

$$y = a(b^x)$$

Start value

multiplier

$$y = a(1+r)^x$$

start value

% rate as a decimal

100%

TABLE

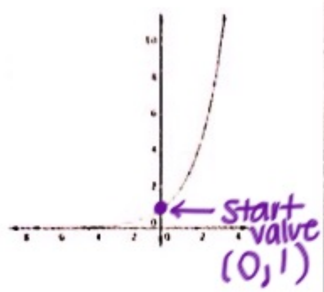
$y = 1(2)^x$

starting

multiplier

X	Y
0	1
1	2
2	4
3	8
4	16
5	32

GRAPH



WORDS

A certain type of bacteria triples every two hours. Suppose you start with 50 bacteria. Write an equation.

multiplier

start

$$y = 50(3^x)$$