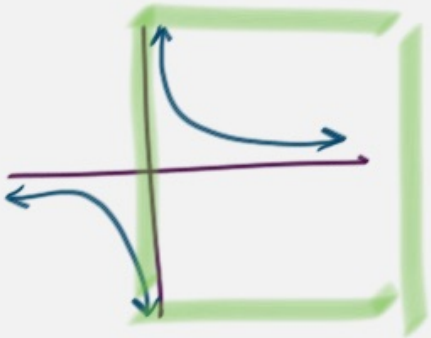
Math 2

Quiz Wednesday

↳ Inverse Variation



decreasing at a decreasing rate

$y = \frac{K}{x}$ K = constant of proportionality. (#)

Investigation: Creating and Solving Inverse Variation Equations

Two variables are inversely proportional if as one variable increases the other variable decreases and their product remains the same. This means for two variables x and y that $xy=k$ and that k is the constant of proportionality. An inverse variation function of the form $f(x) = \frac{k}{x}$ with $k \neq 0$ can be used to model situations where the two variables are inversely proportional. These functions can be used to solve problems.

In this investigation, look for answers to the following question

How can I create and solve inverse variations equations in order to solve problems?

Train Travel

Trains, while not the main mode of transportation that they once were, are still used throughout Europe and North America. An aspect of train travel that is important to train companies and travelers is the consistency of times. For a person that travels by train for work, it is extremely important that the train depart and arrive on time consistently.

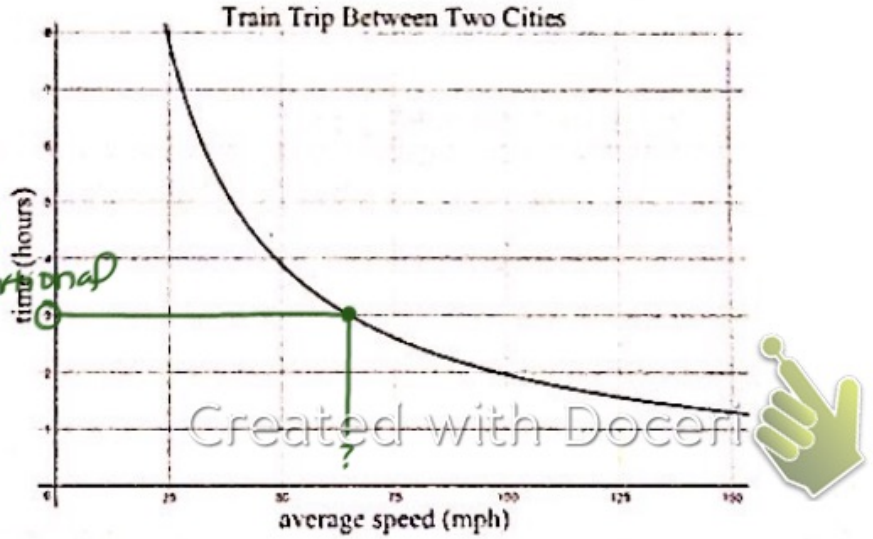
1. Examine the graph on the right that shows the average speed and time it takes to travel between two cities.
   a. Describe the relationship between average speed and time.

graph → decreasing @ a decreasing rate

variables → avg speed is inversely proportional to time

   b. Estimate the average speed needed to make the trip between the two cities in 3 hours.

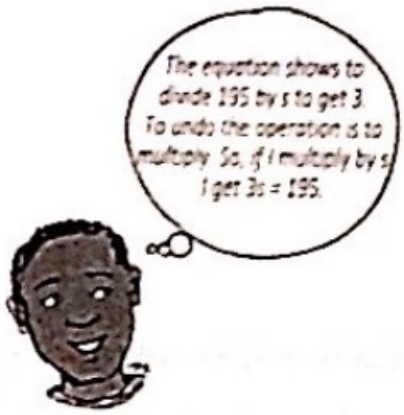
≈ 65 - 70 mph



2. The relationship between the average speed and the time to travel between the two cities can be modeled by the function rule $t(s) = \frac{195}{s}$ → Cop; t → time; s → speed

a. Explain how the equation $3 = \frac{195}{s}$ represents the question from 1b.

t(s) represents time it takes to get to the city, in this case it's 3 hrs.



b. Solve the equation algebraically. Be prepared to share your reasoning. How does it compare to your estimate?

$s \cdot 3 = \frac{195}{s} \cdot s$ $\quad \frac{3s}{3} = \frac{195}{3}$ $\quad \boxed{s = 65}$

c. Use the function rule $t(s) = \frac{195}{s}$ to create and solve equations that answer each of the following questions.

i. What average speed is needed if the train had to make the trip in 2 hours?

$s \cdot 2 = \frac{195}{s} \cdot s$ $\quad \frac{2s}{2} = \frac{195}{2}$ $\quad \boxed{s = 97.5}$

ii. The train took 5 hours to travel between the two cities. What was the average speed?

$s \cdot 5 = \frac{195}{s} \cdot s$ $\quad \frac{5s}{5} = \frac{195}{5}$ $\quad \boxed{s = 39}$

iii. The train arrived an hour late. The trip was supposed to take 2.5 hours. What was the difference in the average speed?

$2.5 = \frac{195}{s} \rightarrow 78$

$3.5 = \frac{195}{s} \rightarrow 55.7$ } = 22.3 mph

Solving Inverse Variation Equations

3. Solve each of the following equations using algebraic reasoning. Check your answers.

a. $x \cdot 19 = \frac{114}{x} \cdot x$

$\frac{19x}{19} = \frac{114}{19}$

$x = 6$

b. $x \cdot 34 = \frac{153}{x} \cdot x$

$\frac{34x}{34} = \frac{153}{34}$

$x = 4.5$

c. $x \cdot -8 = \frac{-44}{x} \cdot x$

$\frac{-8x}{-8} = \frac{-44}{-8}$

$x = 5.5$

d. Since inverse variation equations have a similar structure, the same reasoning is used repeatedly to solve them. Using the general form of $y = \frac{k}{x}$ and describe how to solve the equation for x.

multiply by x, divide by y. $x \cdot y = \frac{k}{x} \cdot x$

$\frac{xy}{y} = \frac{k}{y} \rightarrow x = \frac{k}{y}$

4. Consider the equation $\frac{2}{x} = x + 1$. $y = \frac{k}{x}$

a. How is it similar to and different from an inverse variation equation?

$\frac{k}{x}$ → same; add a constant

b. Solve the equation. Show your work and be prepared to share your reasoning with others.

$\frac{2}{x} = (x+1) \cdot x$

$2 = x^2 + x$

$-2 \quad -2$

$x^2 + 1x - 2 = 0$

	x	2
x	x^2	$2x$
-1	$-1x$	-2

II + III = 1

I · IV = −2

2 · −1

$(x+2)(x-1)$

$x+2=0 \quad x-1=0$

$x = -2 \quad x = 1$

5. Solve each of the following equations using algebraic reasoning. Check your answers.

a. $\frac{24}{x} = (x-2)\cdot x$

$24 = x^2 - 2x$

$-24 \quad -24$

$x^2 - 2x - 24 = 0$

	x	-6
x	x^2	$-6x$
$+4$	$+4x$	-24

II + III = -2

I · IV = -24

$12 \quad 2$

$24 \quad 1$

$8 \quad 3$

$-6 \quad +4$

$(x+4)(x-6) = 0$

$x+4=0 \quad x-6=0$

$x=-4 \quad x=6$

b. $\frac{4}{x} = (3x+1)\cdot x$

$4 = 3x^2 + x$

$-4 \quad -4$

$3x^2 + x - 4 = 0$

	x	-1
$3x$	$3x^2$	$-3x$
$+4$	$+4x$	-4

II + III = 1

I · IV = -12

$6 \quad 2$

$-3 \quad +4$

$1 \quad 12$

$(x-1)(3x+4) = 0$

$x-1=0 \quad 3x+4=0$

$x=1$

$-4 \quad -4$

$\frac{3x}{3} = \frac{-4}{3}$

$x = \frac{-4}{3}$

c. $\frac{-25}{x} = (x+10)\cdot x$

$-25 = x^2 + 10x$

$+25 \quad +25$

$x^2 + 10x + 25 = 0$

	x	$+5$
x	x^2	$+5x$
$+5$	$+5x$	25

II + III = 10

I · IV = 25

$+5 \quad +5$

$1 \quad 25$

$(x+5)(x+5) = 0$

$x+5=0 \quad x+5=0$

$x=-5$

Check Your Understanding $y = \frac{k}{x}$ y is inversely prop. to x.

1. A company has found that the number of its boxes of chocolates sold s is inversely proportional to the price p. When the price is $5, the demand is 800 boxes.

   a. Describe how the number of boxes sold is related to the price. Be sure to focus on the given information that boxes sold is inversely proportional to the price.

   $S = \frac{k}{p} \rightarrow 800 = \frac{k}{5}$

   b. Determine the constant of proportionality and explain what it means in context.

   c. Write a function rule to represent the number of boxes sold as a function of the price.

   d. Create an equation and solve it to answer the question.

   The company wants to sell 2500 boxes. What should be the price of the box of chocola

2. Solve each of the following equations. Show your work and check your solutions.

a. $\frac{120}{x} = 15$

b. $\frac{51}{x} = 17$

c. $\frac{2}{x} = -x - 4$

d. $\frac{x-1}{x} = 2x - 6$