Math 1

Counting in Tree Graphs

In the popular book and movie *Pay It Forward*, 12-year-old Trevor McKinney gets a challenging assignment from his social studies teacher.

Think of an idea for world change, and put it into practice!

Trevor came up with an idea that fascinated his mother, his teacher, and his classmates. He suggested that he would do something really good for 3 people. Then when they ask how they can pay him back for the good deeds, he would tell them to "pay it forward" – each doing something good for 3 other people.

Trevor figures that those three people would do something good for a total of 9 others. Those 9 would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all over the world.

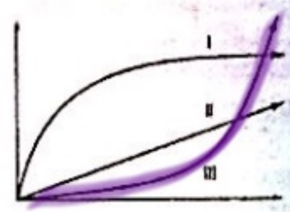
Think About This Situation

Continue Trevor's kind of Pay It Forward thinking.

(a) How many people would receive a Pay It Forward good deed at each of the next several stages of the process?

(b) What is your best guess about the number of people who would receive Pay It Forward good deeds at the tenth stage of the process?

(c) Which of the graphs above do you think is most likely to represent the pattern by which the number of people receiving Pay It Forward good deeds increases as the process continues over time?



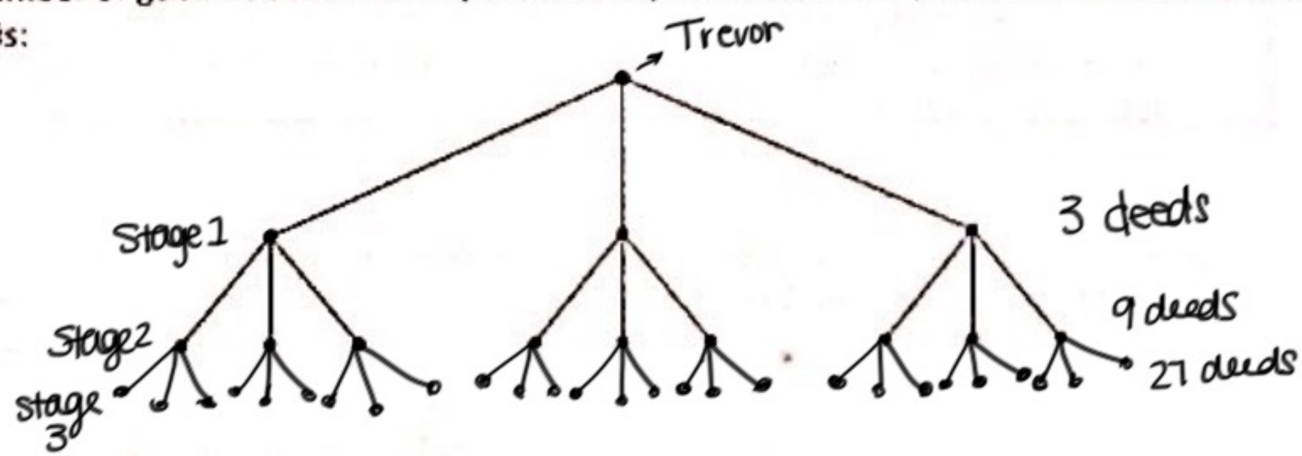
Stage	0	1	2	3	4	5
a.	1	3	9	27	81	243

b. $3^{10} = 59{,}049$

c. III

In this lesson, you will discover answers to questions like these and find strategies for analyzing patterns of change called *exponential growth*. You will also discover some basic properties of exponents that allow you to write exponential expression in useful equivalent forms.

The number of good deeds in the Pay It Forward pattern can be represented by a *tree graph* that starts like this:



The vertices represent the people who receive and do good deeds. Each edge represents a good deed done by one person for another. As you work on the problems of this investigation, look for answers to these questions:

What are the basic patterns of exponential growth in variations of the Pay It Forward process?

How can those patterns be expressed with symbolic rules?

1 At the start of the Pay It Forward process, only one person does good deeds—for three new people. In the next stage, the three new people each do good things for three more new people. In the next stage, nine people each do good things for three more new people, and so on, with no person receiving more than one good deed.

a. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the (*stage, number of good deeds*) data.

Stage of Process	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	3	9	27							

b. How does the number of good deeds at each stage grow as the tree progresses? How is that pattern of change shown in the plot of the data? the stages were multiplied by 3 each time
Points on the graph were getting 3x farther apart

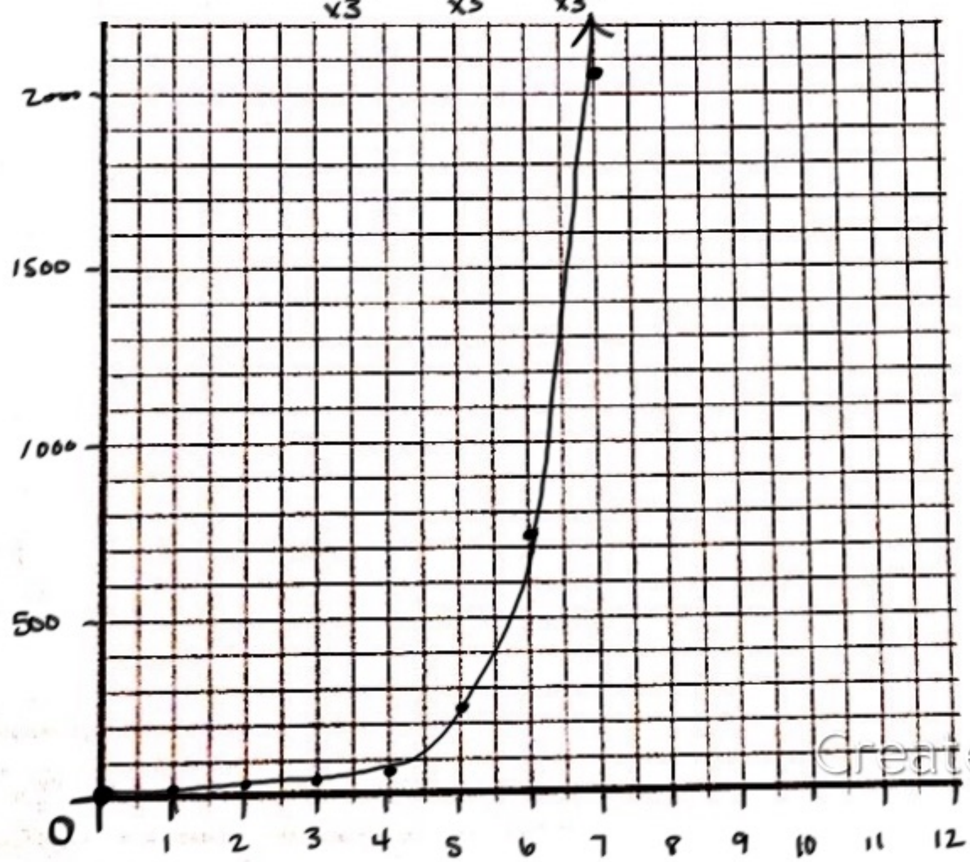
c. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done?

Stage 10

Problem 1a) One person does good deeds for 3 new people

Stage of Process	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	3	9	27	81	243	729	2187	6561	19,683	59,049

×3 ×3 ×3



$y = 3^x$ ← Stage of Process

$3^1 = 3$ ✓

$3^2 = 9$ ✓

$3^3 = 27$ ✓

$3^4 = 81$ ✓

$N = 3x$

$3(1) = 3$ ✓

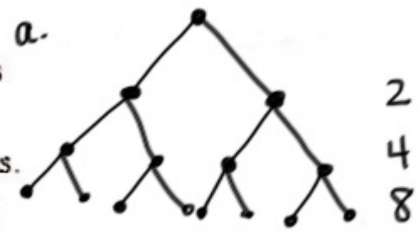
$3(2) = 6$ ✗

2 Consider now how the number of good deeds would grow if each person touched by the Pay It Forward process were to do good deeds for only two other new people, instead of three.

a. Make a tree graph for several stages of this Pay It Forward process.

b. Make a table showing the number of good deeds done at each of the first 10 stages of the process and plot those sample (*stage, number of good deeds*) values.

a.



Tree Graph

Table

Stage:	1	2	3	4	5	6	7	8	9	10
# deeds:										

c. How does the number of good deeds increase as the Pay It Forward process progresses in stages? How is that pattern of change shown in the plot of the data?

multiplied by 2 every stage

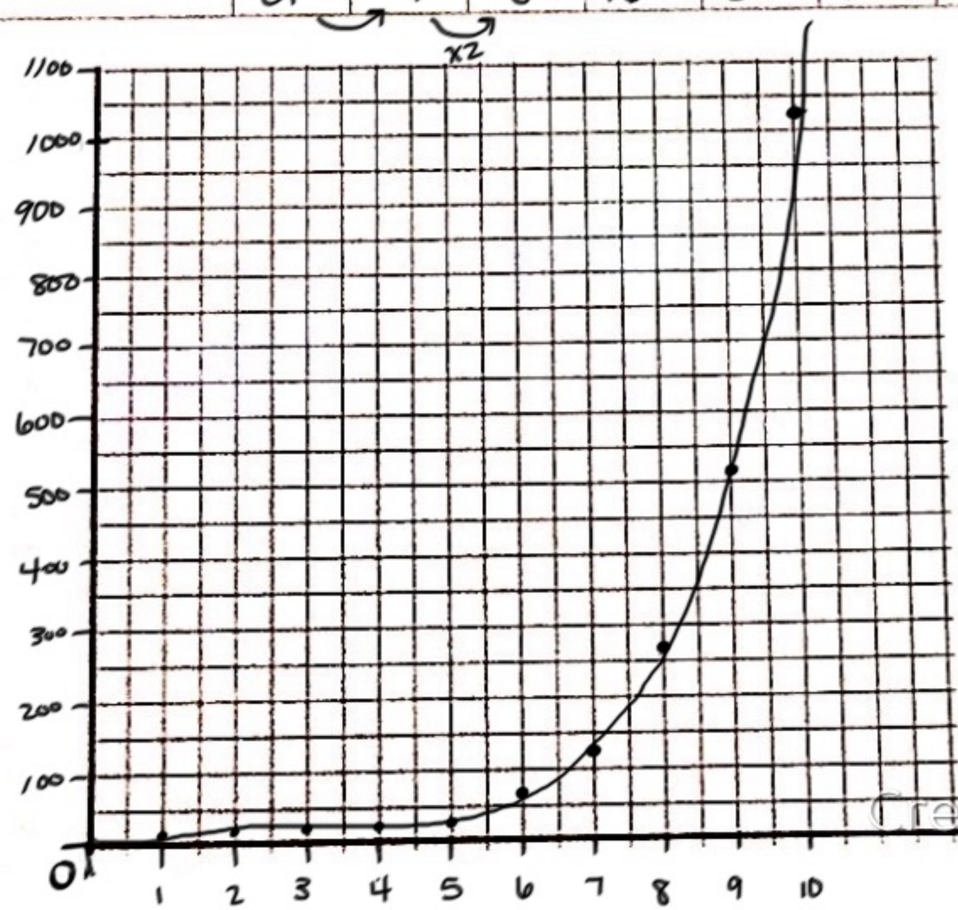
every point on the graph is doubled from the previous one

d. How many stages of this process will be needed before a total of 25,000 good deeds will have been done?

15th stage 2^{15} $2^x = 25{,}000$

Problem 2b) One person does good deeds for 2 new people.

Stage of Process	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	2	4	8	16	32	64	128	256	512	1024



3 In the two versions of Pay It Forward that you have studied, you can use the number of good deeds at one stage to calculate the number at the next stage.

a. Use the words *NOW* and *NEXT* to write rules that express the two patterns.

b. How do the numbers and calculations indicated in the rules express the patterns of change in tables of (*stage, number of good deeds*) data?

c. Write a rule relating *NOW* and *NEXT* that could be used to model a Pay It Forward process in which each person does good deeds for four other new people. What pattern of change would you expect to see in a table of (*stage, number of good deeds*) data for this Pay It Forward process?

a. Next = 3·Now
Next = 2·Now

b. that's how much we multiplied every Previous # by.

c. Next = 4·Now
the table is going to increase faster by multiplying by 4

5 It is also convenient to have rules that will give the number of good deeds *N* at any stage *x* of the Pay It Forward process, without finding all the numbers along the way to stage *x*. When students in one class were given the task of finding such a rule for the process in which each person does three good deeds for others, they came up with four different ideas:

$$N = 3x$$
$$N = x + 3$$
$$N = 3^x$$
$$N = 3x + 1$$

a. Are any of these rules for predicting the number of good deeds *N* correct? How do you know?

b. How can you be sure that the numbers and calculations expressed in the correct "*N* = ..." rule will produce the same results as the *NOW-NEXT* rule you developed in Problem 3?

c. Write an "*N* = ..." rule that would show the number of good deeds at stage number *x* if each person in the process does good deeds for two others.

d. Write an "*N* = ..." rule that gives the number of good deeds at stage *x* if each person in the process does good deeds for four others.

a. $N = 3^x$: b/c multiplying by 3 everytime

b. Plugged in the x & confirmed the same y.

"plug & chug"

c. $N = 2^x$

d. $N = 4^x$