Math 1

- Hand in HW

Math I Homework   ✓ Check Your Understanding   Name: **Key**

The drug penicillin was discovered by observation of mold growing on biology lab dishes. Suppose a mold begins growing on a lab dish. When first observed, the mold covers 7 cm² of the dish surface, but it appears to double in area every day.

• a. What rules can be used to predict the area of the mold patch 4 days after the first measurement:
  i. using NOW-NEXT form?

   Next = 2Now starting at 7

  ii. using "y = ..." form?

   $y = 7(2^x)$

b. How would each rule in Part a change if the initial mold area was only 3 cm²?

   the initial & coefficient would be 3

c. How would each rule in Part a change if the area of the mold patch increased by a factor of 1.5 every day?

   The base & the coefficient in front of the Now would be 1.5.

On Your Own #1
Imagine a tree that each year grows 3 new branches from the end of each existing branch. Assuming that your tree is a single stem when it is planted:

a. How many new branches would you expect to appear in the first year of new growth? How about in the second year of growth?

   year 1: 3   year 2: 9

b. Write a rule that relates the number of new branches B to the year of growth R.

   $B = 3^R$

c. In what year will the number of new branches first be greater than 15,000?

   $15,000 = 3^R$ R = 9

On Your Own #5
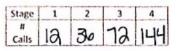Suppose that the Silver Spring Soccer Club has a meeting of the four club directors to decide on whether or not to cancel a scheduled game. Then the directors each start a branch of a calling tree by calling three families, and each of those families then calls three more families. This process continues until all 750 families are contacted.

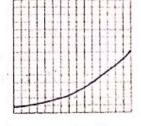a. Sketch a tree graph that shows how the number of people called grows in stages from the first call by the club directors

Stage	1	2	3	4
# Calls	12	36	72	144

c. Write two rules that can be used to calculate the number of calls made at various stages of this calling tree, one in NOW-NEXT form and another in "y = ..." form

   Next = 3Now starting at 12

   $y = 4(3^x)$

d. How many stages of the calling tree will be needed before all 750 families are contacted?

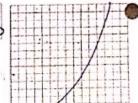 $750 = 4(3^x)$ $3^x = 187.5$ After 5th stage

On Your Own #6
Suppose 50 E. coli bacteria are introduced into some food as it's being processed, and the bacteria begin doubling every 10 minutes.

a. Make a table and a graph showing the number of bacteria from Stage 0 to Stage 6 of the infection.

Stage	0	1	2	3	4	5	6
# Bacteria	50	100	200	400	800	1600	3200

b. Write two rules that can be used to calculate the number of bacteria infecting the food at various stages of this process — one in NOW-NEXT form and another in "y = ..." form

   Next = 2Now starting at 50

   $y = 50(2^x)$

c. Predict the number of bacteria present after 3 hours. Explain how you made your prediction.

   6 → 10 min in 1 hr.

   3 hours → 18 → 0 min

   $y = 50(2^{18}) →$

   $y = 13,107,200$

Investigation 5.1.3 Compound Interest

Every now and then you may hear about somebody winning a big payoff in a state lottery. The winnings can be 1, 2, 5, or even 100 million dollars. The big money wins are usually paid off in annual installments for about 20 years. But some smaller prizes are paid at once. How would you react if this news report were actually about you?

Kalamazoo Teen Wins Big Lottery Prize

A Kalamazoo teenager has just won the daily lottery from a Michigan lottery ticket that she got as a birthday gift from her uncle. In a new lottery payoff scheme, the teen has two payoff choices.

One option is to receive a single $10,000 payment now.

In the other plan, the lottery promises a single payment of $20,000 ten years from now.

① Imagine that you had just won that Michigan lottery prize.

a. Discuss with others your thinking on which of the two payoff methods to choose.

b. Suppose a local bank called and said you could invest your $10,000 payment in a special 10 year certificate of deposit (CD), earning 8% interest compounded yearly. How would this affect your choice of payoff method?

As you work on the problems of this investigation, look for answers to the question

    How can you represent and reason about functions involved in investments paying compound interest?

Of the two lottery payoff methods, one has a value of $20,000 at the end of 10 years. The value (in 10 years) of receiving the $10,000 payoff now and putting it in a 10-year certificate of deposit paying 8% interest compounded annually is not so obvious.

- After one year, your balance will be:
  - $10,000 + (0.08 \times 10,000) = 1.08 \times 10,000 = \$10,800$
- After the second year, your balance will be:
  - $10,800 + (0.08 \times 10,800) = 1.08 \times 10,800 = \$11,664$

During the next year, the CD balance will increase in the same way, starting from $11,664 and so on.

$$y = a\,(b^x)$$

$\uparrow$ Starting value

$\nwarrow$ multiplier

② Write rules that will allow you to calculate the balance of this certificate of deposit.

a. for the next year, using the balance from the year before.

$$\text{Next} = 1.08 \cdot \text{Now} \quad \text{starting at } 10{,}000$$

b. after any number of years x.

$$y = 10{,}000\,(1.08^x)$$

③ Use the rules from Problem 2 to determine the value of the certificate of deposit after 10 years. Then decide which 10-year plan will result in more money and how much more money that plan will provide.

$x = 10 \text{ years}$

$$10{,}000\,(1.08^{10})$$

$$\$21{,}589$$

④ Look for an explanation of your conclusion in Problem 3 by answering these questions about the potential value of the CD paying 8% interest compounded yearly.

a. Describe the pattern of growth in the CD balance as time passes.

b. Why isn't the change in the CD balance the same each year?

c. How is the pattern of increase in CD balance shown in the shape of a graph for the function relating CD balance to time?

d. How could the pattern of increase have been predicted by thinking about the rules (NOW-NEXT and "y = ...") relating CD balance to time?

a. Start off increasing slowly, then it quicker

b. Add in the interest every single year.

c.

d. Since multiplier is 1.08, it will not increase as quicky

⑤ Suppose that the prize winner decided to leave the money in the CD earning 8% interest for more than 10 years. Use tables or graphs to estimate solutions for the following equations and inequalities. In each case, be prepared to explain what the solution tells about the growth of a $10,000 investment that earns 8% interest compounded annually.

a. $y =$ $10,000(1.08^x) = 25,000$

b. $10,000(1.08^x) = 37,000$

c. $10,000(1.08^x) = 50,000$

d. $10,000(1.08^x) \geq 25,000$

e. $10,000(1.08^x) \leq 30,000$

f. $10,000(1.08^x) = 10,000$

a. $X = 12$

b. $X = 17$

c. $X = 21$

d. $X \geq 12$

e. $X \leq 14$

f. $X = 0$

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(6) Compare the pattern of change and the final account balance for the plan that invests $10,000 in a CD that earns 8% interest compounded annually over 10 years to those for the following possible savings plans over 10 years. Write a summary of your findings.

a. Initial investment of $15,000 earning only 4% annual interest compounded yearly $y = 15,000\,(1 + .04)^x \Rightarrow 15,000\,(1.04)^x$

b. Initial investment of $5,000 earning 12% annual interest compounded yearly

$$\overset{.12}{}$$

$$y = 5000\,(1 + .12)^x \Rightarrow y = 5000\,(1.12)^x$$

a. $15000\,(1.04)^{10} = \$22,203.66$

b. $5000\,(1.12)^{10} = \$15,529.24$

$$y = a\,(1 + r)^x$$
$$\overset{\nearrow 100\%}{}$$
Start %rate as decimal

✔ Check Your Understanding

In solving change-over-time problems in Unit 1, you discovered that the world population and populations of individual countries grow in much the same pattern as money earning interest in a bank. For example, you used data like the following to predict population growth in two countries.

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- Brazil is the most populous country in South America. In 2005, its population was about 186 million. It was growing at a rate of about 1.1% per year.

- Nigeria is the most populous country in Africa. Its 2005 population was about 129 million. It was growing at a rate of about 2.4% per year.

a. Assuming that these growth rates continue, write function rules to predict the populations of these countries for any number of years x in the future.

b. Compare the patterns of growth expected in each country for the next 20 years. Use tables and graphs of (*year since 2005, population*) values to illustrate the similarities and differences you notice.

c. Write and solve equations that give estimates when:

   i. Brazil's population might reach 300 million.

   ii. Nigeria's population might reach 200 million.

d. Assuming these growth patterns continue, estimate when the population of Nigeria will be greater than the population of Brazil.